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Developed Capillary Turbulence on the Surface of Normal and Superfluid ^{4}He

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Abstract The capillary turbulence on the surface of normal and superfluid liquid ^{4}He has been studied experimentally. It is observed for the first time that the value of the high-frequency boundary ω_b of the inertial interval increases significantly when liquid helium transits from normal to superfluid state, and that in superfluid He-II the correlation function of the surface deviation from equilibrium state in frequency representation can be described well by a power dependence with the index m close to -4.3.

Keywords Capillary turbulence · Superfluid helium

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1 Introduction

Developed capillary turbulence on the surface of a liquid (Kolmogorov-like cascade) is formed in so-called "inertial interval" of frequency space. Inertial interval is restricted at low frequencies by characteristic frequencies of the external pumping and at high frequencies by a cut-off frequency ω_b where the mechanism of nonlinear energy transfer is changing to the viscous damping of waves [1]. Inside inertial interval the correlation function of the surface deviation (turbulent cascade) is described by the power dependence $I_\omega \sim \omega^m$. In accordance with the theory the value of index m is defined by a spectral characteristic of the driving force: $m = -21/6$ for narrow band drive and $m = -17/6$ for wide band noise pumping.

In the case of a narrow band driving force, the evolution of high-frequency boundary ω_b (with varying excitation amplitude, viscosity, or surface tension coefficient of

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the liquid) can be described by the relation

$$\omega_b \sim A_p^{4/3} \nu^{-2/3} (\sigma/\rho)^{-4/9} \omega_p^{23/9}, \qquad (1)$$

where A_p is the wave amplitude at the frequency of pumping ω_p, ν is the viscosity of the liquid, σ is the surface tension coefficient, ρ is the liquid density. The amplitude dependence of ω_b in (1) was demonstrated by our experiments with capillary waves on the surface of liquid hydrogen [2].

Here we report the results of our experiments on capillary waves on the surface of liquid ^{4}He where the coefficient of viscosity is changed significantly while helium is undergoing the superfluid-to-normal phase transition.

2 Experimental Details

The measurements were made in an optical cell placed inside a helium cryostat. Helium was condensed into a copper cup, placed inside the cell. The inner diameter of the cup was 30 mm and its height was 4 mm. Above the cup a copper plate was arranged at a distance of 3 mm from the liquid surface. At the bottom of the cup a β-radioactive plate was placed to generate positive and negative charges. A two-dimensional layer of positive charges (snowballs) was created immediately below the liquid surface under the action of dc voltage $U \approx 800$ V applied between the cup and the top capacitor plate. We performed measurements on the surface of liquid He-I at the temperature $T = 2.3$ K, and on the surface of liquid He-II at $T = 1.8$ K and 1.95 K.

The waves on the charged surface were excited by an ac driving voltage with the amplitude 10–100 V, applied between the cup and the top plate in addition to dc voltage. The surface oscillations determined the variation of the total power of a laser beam reflected from the liquid $P(t)$. The variation of $P(t)$ was measured with a photodetector and sampled with a 16-bit analog-to-digital converter. The correlation function I_ω was obtained by calculating a Fourier transform of the detected signal $P(t)$. In our experiments we worked with a broad laser beam (the diameter of the light spot was more than the length of the surface waves excited). So, in accordance with the results of our previous investigations of capillary waves on the surface of liquid hydrogen, the correlation function $I_\omega \sim P_\omega^2$ in a frequency range higher than 10 Hz.

The frequency of the driving force in our experiments was more than 10 Hz. That suggested that we were working with capillary waves, but not with gravity ones. The amplitude of the wave at the frequency of excitation was estimated from the maximum angle of deviation of the reflected laser beam from the surface of liquid helium. The maximum value of the angle (0.05 rad) was limited by the diameters of optical windows of the cryostat.

3 Results

The spectrum of surface oscillations P_ω^2 of superfluid He-II at $T = 1.8$ K is shown in Fig. 1. The surface was excited by the harmonic force at a frequency $\omega_p/2\pi =$

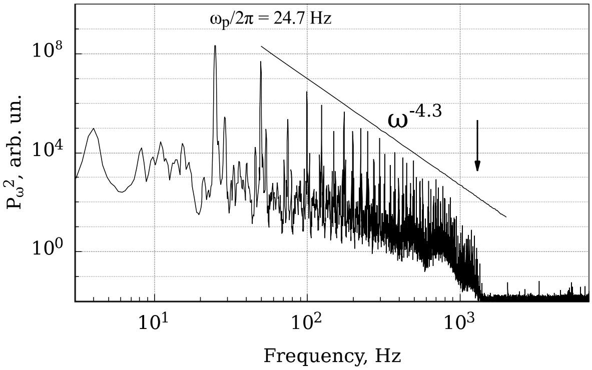

Fig. 1 The spectrum of surface oscillations P_ω^2 on the surface of superfluid He-II at $T = 1.8$ K. Averaging over 32 files of 3 s duration. Excitation frequency is 24.7 Hz. *Straight line* corresponds to a power law function $P_\omega^2 \sim \omega^{-4.3}$. The *arrow* marks the position of the high-frequency boundary ω_b

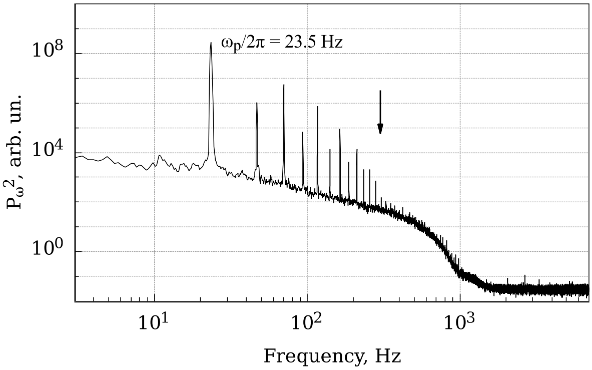

Fig. 2 The spectrum P_ω^2 on the surface of normal He-I at $T = 2.3$ K. Averaging over 32 files of 3 s duration. The *arrow* marks the position of the high-frequency boundary ω_b. At $T = 2.3$ K ω_b is smaller than that at $T = 1.8$ K (Fig. 1). The smooth background extending from low frequencies to 1 kHz is assumed to be a consequence of boiling of the normal liquid

24.7 Hz. The amplitude of wave at the frequency of excitation was close to 0.05 mm, about a half of the highest admissible one in geometry of our experiment. With this amplitude of the driving force we could observe the high-frequency boundary. The high-frequency boundary $\omega_b/2\pi$ of the inertial interval arrowed in Fig. 1 is equal to 1000 ± 100 Hz.

The distribution P_ω^2 of capillary waves on the surface of He-I at $T = 2.3$ K with almost the same frequency and amplitude of the driving force is shown in Fig. 2. One can distinguish about 15 peaks only in the distribution. The position of the high-frequency boundary $\omega_b/2\pi = 300 \pm 100$ Hz is marked by the vertical arrow. During the experiment the temperature of the cell was lowered slowly to decrease the intensity of boiling of the normal liquid. It is assumed that the consequence of that non-intensive boiling is seen in Fig. 2 as a smooth background extending from low frequencies to 1 kHz.

The spectrum of well developed turbulence on the surface of superfluid He-II at $T = 1.95$ K is shown in Fig. 3. The surface oscillations were excited by a harmonic force at frequency $\omega_p/2\pi = 13.6$ Hz. The wave amplitude at the frequency of excitation was close to the highest admissible one, i.e. about 0.1 mm. The frequency dependence of the correlation function $I_\omega \sim P_\omega^2$ can be fitted by a power function of frequency ω^m with the index $m = -4.3$. The averaged value of the index obtained over several different experiments at the same temperature is close to -4.0 ± 0.3.

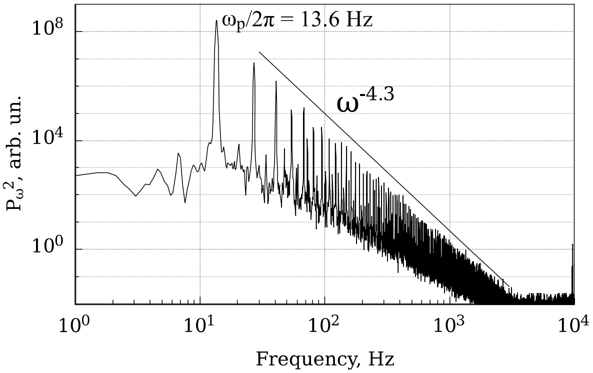

Fig. 3 Well developed turbulent cascade on the surface of He-II at $T = 1.95$. Averaging over 16 files of 3 s duration. *Straight line* corresponds to a power law function $P_\omega^2 \sim \omega^{-4.3}$

Table 1 Properties of liquid helium

Temperature, K	1.8	1.95	2.3
Density ρ, g/cm^3	0.1453	0.1455	0.1458
Surface tension σ, dyn/cm	0.316	0.306	0.277
Kinematic viscosity ν, cm^2/s	0.000089	0.000096	0.0002

The observed value of index m differs from theoretical predictions and previous experiments with hydrogen. The reasons for this discrepancy necessitate further investigations.

The coefficients of kinematic viscosity and surface tension of liquid helium are changed significantly with lowering the temperature from 2.3 K to 1.8 K. The properties of liquid helium at three temperatures are given in Table 1 [4].

Estimation of ratio of the high-frequency boundary $\omega_{1.8\text{K}}$ at $T = 1.8$ K to the high-frequency boundary $\omega_{2.3\text{K}}$ at $T = 2.3$ K in the frames of Zakharov model (1) gives the value $\omega_{1.8\text{K}}/\omega_{2.3\text{K}} = 2.1$. The experimental value of the ratio is equal to 2.2 ± 0.4. Thus, formula (1) is in agreement with experimental observations of high-frequency boundary variation with changing both the amplitude of exciting force and the viscosity of liquid.

4 Conclusions

The use of liquid helium allowed us to observe for the first time a strong shift of the high-frequency boundary of inertial interval when the viscosity of the liquid was changed by the increasing of the temperature. This shift can be described in the frame of Zakharov theory [1] for capillary turbulence excited by a harmonic force. However the observed index of power law function describing the dependence of correlation function I_ω on frequency differs from the theoretical prediction $m = -21/6$. It should be mentioned that in our experiments with liquid hydrogen [3] the power index $m = -3.7 \pm 0.3$ was close to the value predicted by theoretical consideration [1]. We hope to determine the reasons for the discrepancy in future investigations.

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